Conceptual Design and Analysis of an MXER Tether Boost Station

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Momentum-exchange / electrodynamic reboost (MXER) tether systems show great promise for use in propellantless orbital transfer. In 1998, MSFC and Boeing conducted a simple, preliminary examination of the system requirements of a tether facility to boost payloads from LEO to GTO. Work conducted at MSFC and TUI over the last two years has updated and refined these results, and led to alternate configurations and concepts that show greater promise for successful utilization. Two appendices are included that detail analysis techniques and mathematical derivations that can be used in tether facility design.

Introduction

The development of tether technology has opened up an exciting new possibility for spacecraft—propellantless propulsion. Rockets push against their own exhaust, but an electrodynamic tether pushes against the Earth's magnetic field, and in essence, the Earth itself, to enable payloads to acquire higher-energy orbits.

A pure momentum-exchange (MX) tether does not create orbital energy; it only exchanges it. If it catches and throws a payload, its orbital energy will be reduced, and it will assume a lower orbit. Without reboost, it will soon lose too much orbital energy and enter the atmosphere and burn up.

Any type of propulsion system, in theory, could be used to reboost an MX tether. Chemical, nuclear, and electric are all options, but if any *rocket* reboost technologies are chosen, the MX tether will have a payload fraction that is governed by the specific impulse of the propulsion system, according to the rocket equation.

On the other hand, a pure electrodynamic (ED) tether is limited to the regions above the Earth where the ionosphere and magnetic field are relatively strong (<1000 km). It collects electrons from the ionosphere to flow current through its conductive tether[1]. That tether acts like a wire moving through the field lines of the Earth's magnetic field; consequently a **J x B** force is exerted on the system. The ED tether can passively generate power (at the expense of orbital energy) or use a power supply to drive current through the tether and generate motive force (increasing orbital energy).

In theory, an ED tether could dock with a payload and slowly spiral up to a higher orbit, then release it and spiral back down. However, again the ED tether is limited to altitudes less than 1000 km, and achieves performance similar to other low-thrust, high-power propulsion systems that have very low thrust-to-weight ratios.

The MX and ED tethers, by themselves, do not achieve exceptional improvements in performance over existing technologies, but a hybrid of the two, the momentum-exchange/electrodynamic reboost (MXER) tether, may have capabilities far beyond either technology separately.

In principle, a rotating MXER tether in an elliptical orbit could catch a payload in a low Earth orbit, carry it for a single orbit, and than throw it into a higher energy orbit, all in a short period of time. It can then employ electrodynamic reboost over a period of weeks to restore the orbital energy it gave to the payload. Hence, the tether system can give a payload all of the performance and efficiency of a "high-thrust", impulsive orbit transfer, but slowly reboost itself using only electricity.

1998 Tether Transportation System (TTS) Study

In March 1998, a NASA study[2] was commissioned to assess the viability of a tether facility to boost payloads from low Earth orbit (LEO) to geosynchronous transfer orbit (GTO). The study also included participation from Boeing and the Smithsonian Astrophysical Observatory.

The study considered both single-tether and multiple-tether designs to conduct the boost from LEO to GTO. Ultimately, it concluded that a two-tether design, with one tether in a 400 x 2019 km orbit and the other in a 434 x 25048 km, offered the lowest mass solution. The study also baselined high-Isp electric propulsion for orbital reboost, and thus necessitated refueling operations to replenish the exhausted propellant.

The astrodynamics in the study made the simplifying assumption of a spherical Earth, and thus did not assess the impact of the oblateness of the Earth, and the consequent nodal and apsidal regression of the tether orbits. This effect has dramatic consequences for tether design.

Progress since the TTS Study

Since the TTS study was conducted significant progress has been made at MSFC, Tethers Unlimited[3], and Boeing in the design and analysis of MX tethers.

Astrodynamics

More advanced astrodynamic analyses have been conducted on the MX tether concept. It was quickly seen that the assumption of a spherical Earth had inadvertently led to a nonviable tether design.

The oblate shape of the Earth causes changes in position of the line of nodes and line of apsides of an orbit.[4] Specifically, the line of nodes (the line formed by the intersection of the orbital and equatorial planes) will rotate about the planetary rotational axis; the line of apsides (the line drawn between the apoapsis and periapsis of an orbit) will rotate about the angular momentum vector of the orbit.

Because nodal and apsidal regression are functions of orbital energy, eccentricity, and inclination, two spacecraft that have the same inclination but different eccentricities will soon become non-planar due to differential nodal regression, as illustrated in Figure 1. Hence, a tether system and a payload in a non-equatorial orbit will soon be out-of-plane with each other.

However, when an object is in an equatorial orbit around a planet, the planetary rotational axis and the angular momentum vector of the orbit will be collinear, and nodal and apsidal rotation will become coplanar. Hence, sustained nodal regression will not change the coplanarity of the tether and payload orbits if they are both equatorial.

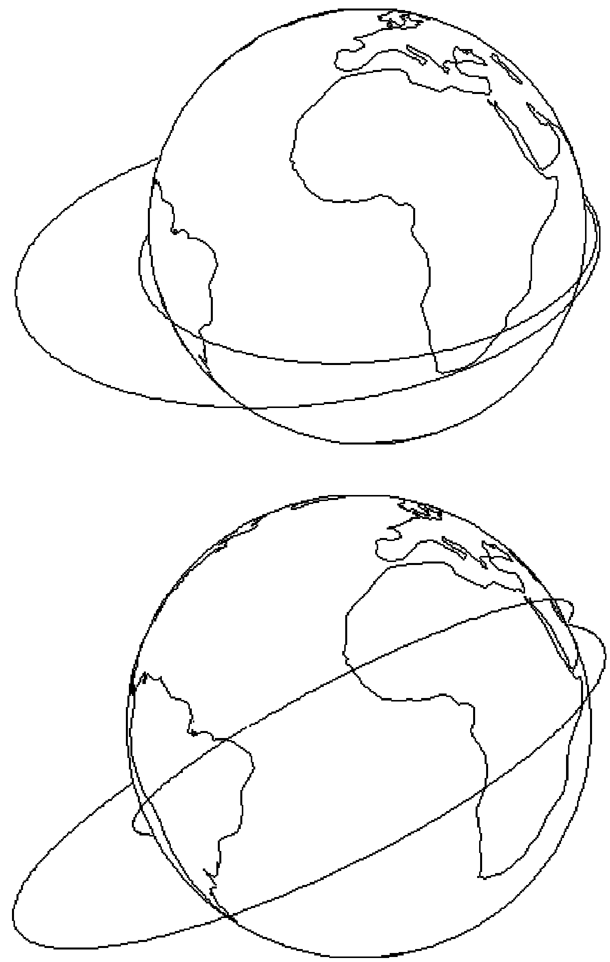

Figure 1: Two non-equatorial orbits, initially coplanar, will evolve differently due to differential nodal regression; 15 days later, they are decidedly non-planar.

While the equatorial orbit cures the problem of differential nodal regression, the differential rates of apsidal regression will soon doom a two-tether system like the one baselined in the TTS study. In that study, the orbital periods of the first and second tethers were configured so that they were integer multiples of each other; hence, if the payload was released by the first tether and missed being caught by the second tether, it would have another opportunity several orbits later. However, when the effect of apsidal regression is factored into the problem, it becomes clear that even for two tethers with resonant periods, two collinear lines-of-apsides will be an exceedingly rare occurrence.

The solution to this problem is to abandon the two-tether system. Although it has mass benefits relative to a single-tether system, **the reality is that it is astrodynamically nonviable**.

Multiple rendezvous opportunities exist for a single-tether system if the payload's orbit is further constrained to be a circular one. That way, the location of tether tip/payload contact can move in angular position around the payload's orbit as the line of apsides of the tether's orbit rotates.

Hence, astrodynamic reality drives the design from a two-tether system in an arbitrary inclination to a single tether in an elliptical equatorial orbit and a payload in a circular equatorial orbit. An extended derivation of the orbital requirements necessary to achieve repeated rendezvous attempts in these orbits is given in Appendix A.

Orbital Reboost

Another important advancement of tether design since the TTS study has been the pursuit of electrodynamic reboost of the tether station as opposed to propulsive reboost.

The TTS study baselined the use of high-Isp ion engines as the propulsive means of restoring orbital energy and angular momentum to the tether station that is lost during catch and throw operations. Use of propulsive reboost limits the performance of the tether station to the mass fractions governed by the rocket equation; although that the tether can deliver these impulses quickly and at an astrodynamically efficient location.

Concomitant with the use of propulsive reboost is the requirement to resupply the tether station with propellant at given intervals. This leads to rendezvous and docking requirements with the tether station that drive the design in directions that may not be ideal from an astrodynamic standpoint. For example, the TTS study settled on a tether configuration that located most of the tether mass at the CM of the tether facility, rather than utilizing that additional mass as endmass. Also, rendezvous and docking with a spinning tether in an elliptical orbit would definitely be beyond the capabilities of the space shuttle, and may require a dedicated transfer vehicle and automated rendezvous and docking capability.

As has been noted in the introduction, electrodynamic reboost, while less mature than propulsive reboost, offers a way to conceivably break the bands of the rocket equation altogether, and offer purely propellantless propulsion. It would involve weaving a length of conductive metallic tether into the structure of the strength tether. The electrodynamic tether would be designed so that its integrated force vector would be directed through the tether's CM.

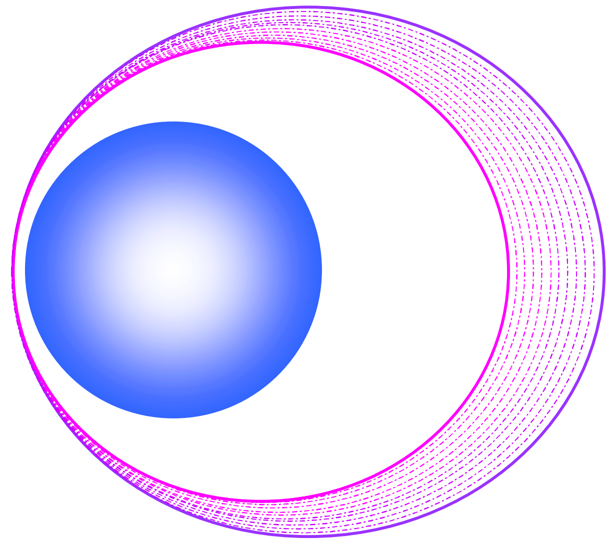

Figure 2: Electrodynamic reboost centered on an arc about perigee can be used to boost apogee and restore orbital energy.

By using solar power to drive electrical current through the electrodynamic tether on an arc centered about perigee, the apogee of the tether station could be reboosted slowly over a period of weeks, as shown in Figure 2. The orbital energy and angular momentum that had been transferred to the payload would be restored to the tether station. Effectively, the electrodynamic tether would move through the Earth's magnetic field in the same way that an armature moves through an magnetic field in an electric motor, and it would "push" against the magnetic field, transferring some of the Earth's rotational angular momentum to the tether station.

For those who might be squeamish about "slowing down the Earth", it should be noted that the amount of the Earth's rotational angular momentum transferred to the tether is *many, many orders of magnitude less* than the amount that is currently being transferred to the Moon's orbit through tidal action!

Electrodynamic tether propulsion is being pursued actively now at MSFC through the ProSEDS mission, which is scheduled to fly in late 2001. ProSEDS will demonstrate passive current flow through the tether that will slow down the orbit of the spacecraft and cause an accelerated reentry. ProSEDS and its follow-on missions will be our first steps in demonstrating that electrodynamic tether propulsion can be used to change orbits.

Tether/Payload Rendezvous

From the outset, it has been recognized that rendezvous between the tip of the tether and the payload *is the most significant problem in momentum-exchange tethers*. Perhaps a bit of perspective may enlighten the difficulty.

When orbital rendezvous takes place today between two spacecraft, they strive to match orbits with one another. In the simplest sense, it could be said that they strive to make their position, velocity, and acceleration vectors equal so that they can dock to one another. If one has ever watched the docking operations of the shuttle and space station, it is quickly seen that these docking operations, as they near their conclusion, take place exceedingly slowly and with extremely high precision, often with millimeters of positional error and millimeters per second of velocity error.

Tether/payload rendezvous is a completely different operation. If the tether and payload were to match orbits by matching position, velocity, and acceleration, there would be no momentum to exchange between them!

Therefore, tether/payload rendezvous is based around the concept of matching the position and velocity of the tether tip and the payload at one point in time and space. Acceleration is NOT matched. This is accomplished by setting the angular rotation rate of the tether such that the tether tip velocity is the difference between the orbital velocity of tether CM and the payload. A simpler way to express this concept is to imagine the tether like a wheel, rolling around the orbit of the payload, and the tip of the tether like a point on the wheel, making contact with the payload for an instant of time.

Another way to imagine tether/payload rendezvous is to imagine working on the roof of a house on a hot Saturday. You call down to someone to throw you up a drink. They do so, and the drink rises quickly while decelerating, and comes to a stop right before your eyes for a moment. It has the same position and velocity as you, but different acceleration. You reach out and grab the drink at that instant or else it will fall back down to the ground. Similarly, from the perspective of the payload, the tether tip descends rapidly from above, comes to a stop, and then ascends rapidly. During that moment of zero relative velocity, contact must be made between the tether tip and payload.

So while conventional orbital rendezvous takes place over a long period of time at very low relative velocity, tether/payload rendezvous must happen nearly instantaneously, and must be tolerant of significant position and velocity errors. An excellent animation of tether/payload rendezvous is available on the Tethers Unlimited website at:

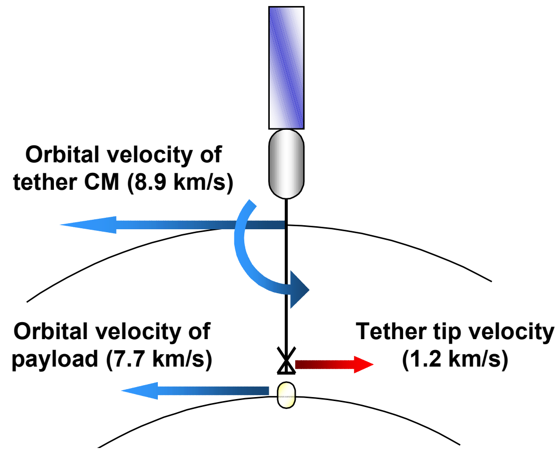

Figure 3: The tip velocity of the tether is configured to be equal to the difference in orbital velocity between the tether and payload, enabling them to match position and velocity instantaneously.

http://www.tethers.com/LEO2GTO.mov

Therefore, using conventional rendezvous techniques for tether rendezvous is of limited utility. They don't work fast enough, with enough error tolerance, to be useful. But it should also be remembered that the requirements for tether/payload are different than for shuttle/station rendezvous. There is no need for a hard docking; they just need to hold on to each other long enough for the tether to transfer significant momentum to the payload before it is released.

The payload capture mechanisms that have been developed at MSFC over the past year are based around this concept. They are tolerant of several meters of positional error and nearly a meter per second of velocity error. They are designed to work very quickly, be very lightweight, and to collapse to a structurally efficient configuration upon payload capture. They are designed to be essentially passive and require no propellant replenishment, power, or active control. They may also have the added benefit of reducing the intensity of the payload capture maneuver on the tension of the tether.

Currently, Tennessee Tech University, under contract to MSFC, is researching the mechanical properties of these payload capture mechanisms and will be conducting preliminary ground testing in the summer of 2001.

Tether Station Configuration

Advancements in the configuration and design of tether boost stations have been made since the TTS study. During the summer of 2000, research into advanced lunar and Mars missions led to an effort to design a tether facility that could accommodate those mission requirements. Several configuration innovations were identified during that design cycle that were later applied to the design of a smaller tether facility for boosting communications satellites from LEO to GTO.

For the lunar/Mars boost station, the following were technology drivers:

- Due to the large initial mass of the station (200 MT), the design had to be modular and capable of increased capacity over its design life.
- All elements were designed to fit within the mass and volume constraints of a hypothetical second-generation launch vehicle. (15m long, 5m diameter cylinder, 20 MT to a 400 x 400 km orbit)
- The station had to be capable of truly propellantless electrodynamic reboost. (i.e. no consumables)
- Due to the relatively short orbital arc through which electrodynamic reboost was feasible, the station needed substantial energy storage capability.
- Access between the payload/tether tip and the central station was highly desirable.

Based on these requirements, an innovative configuration was developed, shown in Figure 4. It consisted of a central, "backbone" ballast tank, that could be filled with water as the station grew in mass. The tank could be launched in a single launch and would serve as the structural hub of the tether station. Its integrated truss structure would accept flywheels (for energy storage) and tether reels, each sized for the capacity of the launch vehicle.

The facility would be designed to be assembled on orbit and have a gross mass of 200 MT at completion, most of which would be the mass of the tether still wound on their spools. Of the eight tether spools, only two would be deployed initially, giving the tether facility an initial payload capacity of 20 MT. This payload capacity would be sufficient to support early human missions to the lunar surface. The remainder of the tether spools would remain undeployed and serve as ballast mass.

It was envisioned that as round-trip traffic between the lunar surface and LEO grew, returning lunar spacecraft would exchange their payloads on the lunar surface for ballast mass, most likely water. This water would be acquired at the tip of the tether and then transported by means of the tether climber. At the central station, this water would be used to fill the ballast tank.

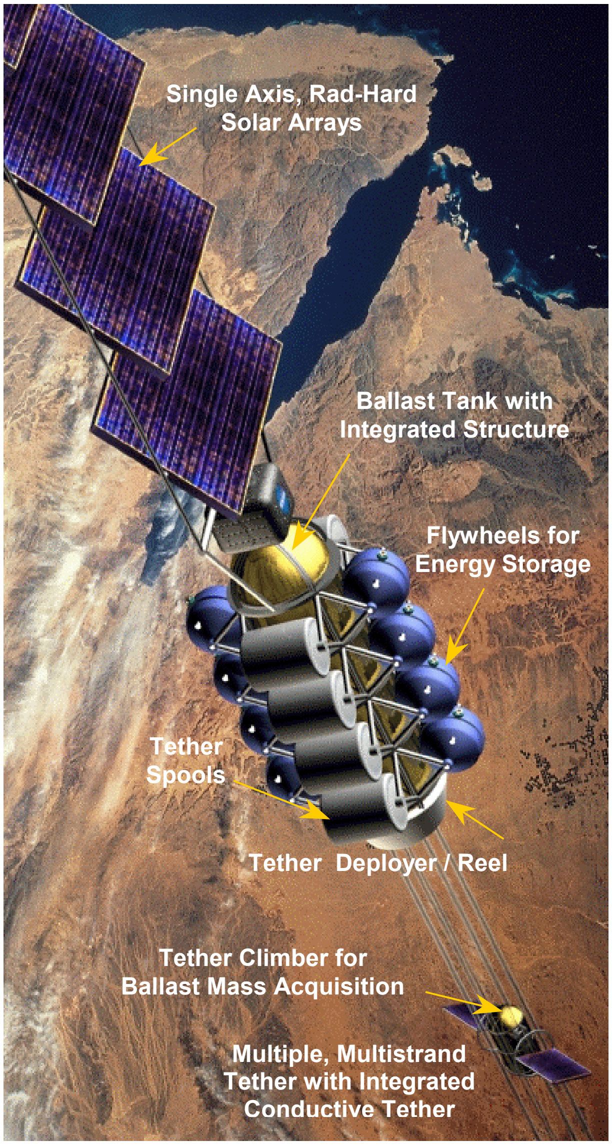

Figure 4: Conceptual configuration of a modular tether station demonstrating power collection, energy storage, staged tether deployment, and allocation for ballast mass collection.

After a number of missions, the ballast tank would reach its full capacity of 200 MT, and the remainder of the tether would be deployed (with the assistance of the tether climber). At this stage, the total mass of the tether facility would reach 600 MT and the station would reach its final payload capacity of 80 MT. This payload capacity would be sufficient to support significant human Mars mission efforts, as well as larger scale lunar exploration.

Conclusion

Significant progress on the astrodynamics, orbital reboost, payload/tether rendezvous, and configuration have been achieved since the MSFC/Boeing Tether Transportation System study of 1998. Significant changes to the baseline design since the TTS study include:

- Single tether facility in an equatorial, elliptical orbit instead of two tether facilities
- Propellantless electrodynamic reboost instead of high specific-impulse electric propulsion
- Design and development of payload catch mechanisms with significant margin for position and velocity error
- A modular, upgradable tether facility configuration that concentrates mass at the endmass rather than the center of mass of the tether system

Acknowledgements

Dr. Robert Hoyt and Dr. Robert Forward of Tethers Unlimited have done tremendous work improving the state of the art in tether design; collaborations with them have been very valuable. Leroy Allen of Media Fusion was an integral part of the design of the tether configuration presented herein and generated all the visualizations. I would like to especially thank Les Johnson, Jonathan Jones, and Charles Schafer of MSFC for support, new ideas, and lots of patience. And thanks to my wife, Quincy Sorensen, for inadvertently inventing the payload catch mechanism.

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Appendix A

Basic Tether Orbital Design Principles

This appendix will detail the basic astrodynamic principles used in the design of a MXER tether facility; it will defer detailed analysis of the electrodynamic reboost to a future paper.

Basic, intermediate, and "advanced" levels of analysis will be presented, as well as a technique for achieving a resonant orbit (multiple catch opportunities) between the payload and the tether while still accounting for the secular effects of Earth's gravitational zonal harmonics (J2, J4, J6). It is hoped that these principles, or variations on them, will be of use to those attempting to design momentum-exchange tether facilities.

Basic Orbital Analysis

Several assumptions are useful when beginning the analysis of an MXER tether. The first assumption is that the tether station has an infinitely massive endmass and a negligible tether mass. This means that the center of mass of the tether/payload system will not shift when payloads are caught and released. This in turn will mean that the rotational moment arm of the tether will not change in length. It will be seen that the movement of the center of mass of the payload/tether system is what governs the momentum exchange inherent in tether orbital transfer.

Another useful assumption is that the tether is always in the proper phase. In other words, when the tether is catching payloads, one can assume it is on the downswing (hanging down relative to the local horizontal), and when the tether is throwing payloads, it is on the upswing (hanging up relative to the local horizontal). Additionally, it is assumed that all these operations take place at the perigee of the tether station's orbit.

As the model becomes more refined, it is possible to remove certain simplifying assumptions (infinite mass) and assure others (proper phasing).

The payload begins in a circular orbit; assume 400 km altitude, for instance. Then assume that the tether is 100 km long. Because of the assumption of infinite endmass, this means that the rotational moment arm on the tether is also 100 km long.

The circular orbital velocity of the payload can be calculated[5] from the following equation (which for 400 km leads to an orbital velocity of 7.67 km/s):

$$v_{circ} = \sqrt{\frac{\mu}{r}}$$

Next, the ΔV required to achieve the desired orbit must be known. Three logical destinations for payloads thrown by a tether would be geosynchronous transfer orbit (GTO) for communications satellites, a 48-hr, pre-escape orbit for planetary spacecraft using high-thrust propulsion, or a trans-lunar injection (TLI) for lunar missions and planetary spacecraft using low-thrust propulsion.

LEO (400x400) to GTO (400x36000 km)	2400 m/s
LEO (400x400) to 48hr (400x120700 km)	2900 m/s
LEO (400x400) to TLI (400x384400 km)	3100 m/s

The desired tether tip velocity will roughly be ½ of the ΔV to be imparted to the payload. For a throw into a trans-lunar trajectory, assume a total ΔV of 3 km/s. Divided into two phases (catch and throw) gives a tether tip velocity of 1.5 km/s.

Since the tether's tip will have to be at the same position and velocity of the payload, it can be assumed that the center of mass (CM) of the tether should be moving 1.5 km/s faster than the payload. Hence, the tether's CM is at an altitude of 500 km (400 km + 100 km) and a velocity of 9.2 km/s (7.7 km/s + 1.5 km/s). These values will yield a preliminary apogee for the tether CM. Note that these values are obviously incorrect, but they are good approximations.

Using the following equations[5]:

$$\varepsilon_{MXER} = \frac{v^2}{2} - \frac{\mu}{r}$$

yields a specific mechanical energy of –16.0 km^2/s^2. Semi-major axis and apogee can then be calculated, and the apogee comes out as 11,620 km altitude.

This apogee value can then serve as an input to the spreadsheet tether model. Experience has shown that it is better to set up the spreadsheet using the following independent variables:

- Circular orbital altitude of the payload
- Tether length
- Apogee of the tether CM

By constraining the perigee of the MXER tether according to the tether length and the payload's orbit, and then varying the apogee so as to target the DV requirement, the system is more easily optimized for other, more subtle constraints.

Acceleration on the payload is a function of the tether tip velocity and the tether length. If the tip acceleration exceeds an acceptable value, the tether length can be increased. This will reduce tip acceleration while maintaining tether tip velocity. Conversely, if tip acceleration is low, the tether length can be reduced in order to reduce the susceptibility of the tether to damage or debris. As will be shown, tether mass is *not* a function of tether length, but of tether tip velocity.

Intermediate Orbital Analysis of Momentum Exchange

In the basic stage of analysis, the tether station loses no altitude for each catch and throw because an infinite ballast mass has been assumed. This can be changed by entering mass values for the endmass and the tether. Now the moment arm of the tether will no longer be the tether length; it will instead be the distance from the CM of the tether station to the payload.

The location of the CM will be critical for further calculations of orbital mechanics, because as the tether catches and throws payloads, the CM moves up and down the length of the tether. The propagation of the position and velocity of the CM is what determines how much the tether orbit drops during catching and throwing maneuvers. This will become more clear as the analysis proceeds.

To begin with, guess a value for the endmass. Typical values are 5-8x the mass of the payload, although no answer is the "right" answer. In theory, tether stations could be defined with no endmass at all!

The next step is to calculate the mass of the tether itself. In an unpublished work, Hans Moravec[7,8] derived the mass of a spinning tether in free space. This equation gives the ratio of the mass of the payload and the mass of the tether as a function of tether tip velocity and tether material:

$$MR = \frac{m_{tether}}{m_{tip}} = \sqrt{\pi}(VR)\exp\left(VR^2\right)erf(VR)$$

In this equation, VR is the velocity ratio and is defined as the tip velocity of the tether divided by the characteristic velocity V_c of the tether material, which is further defined by the following equation.

$$V_c = \sqrt{\frac{2T}{Fd}}$$

T is the tensile strength of the tether material; d is the material density, and F is a safety factor. A safety factor of 3 is assumed in all these calculations. Characteristic velocity can be thought of as a specific tensile strength. Erf is the Gaussian error function.

Further examination of the tether mass ratio equation shows that it is only a function of one variable: the velocity ratio. The exponential dependence of the tether mass on the *square* of the velocity ratio can result in a very large increase in tether mass for a given tip velocity. Hence, it is critical to choose a material with as high a characteristic velocity as possible.

The table below[6] shows that Spectra 2000 has the best characteristic velocity of any material in existence. Spectra 2000 is a form of highly oriented polyethylene manufactured by AlliedSignal. It is currently mass-produced as fishing line, and sold under the brand name Spiderwire®.

Assuming the use of Spectra 2000 as the tether material with a safety factor of 3, the mass ratios between the

material	Vc (km/s)	density	20 C	300 C	600 C	800 C	1000 C	1200 C
Spectra 2000	1.658	970	4.0					
Carbon	1.599	1800	6.9					
Zylon (PBO)	1.574	1560	5.8	3.7				
Textron beta-SiC	1.262	2930	7.0	6.6	6.0	5.6	5.2	4.5
Carbon/Ni-coated	1.222	2680	6.0					
S-glass	1.120	2500	4.7					
Quartz Glass (SiO2)	1.044	2200	3.6	3.6	3.6	3.6	3.6	
0.72 beta-SiC/Ti-coate	0.995	3370	5.0	4.8	4.3	4.0	3.7	3.2
Tyranno (SiTiCO)	0.957	2550	3.5	3.5	3.5	3.5	3.5	3.5
Nextel (alpha-Al2O3)	0.753	3880	3.3					
Altex (Al2O3/SiO2)	0.696	3300	2.4	2.4	2.4	2.4	2.4	1.5
0.65 Nextel/Al-coated	0.560	3400	1.6	1.4				
Tungsten Wire	0.316	19350	2.9	2.9	2.9	2.9	2.9	2.9

tether and payload for the following tip velocities are:

1200 m/s tip velocity (LEO to GTO)	1.50
1500 m/s tip velocity (LEO to TLI)	2.86

Hence, a tether to throw a payload to trans-lunar injection must have nearly 3 times the mass of the payload it is throwing.

Using the center of mass of the tether, the endmass, and the payload, it is possible to calculate a CM for the system, both before and after catching the payload.

Another important realization is that the tether station's rotational rate remains constant during catching and throwing of payloads. This statement seems counterintuitive, but because the payload has the velocity appropriate for its position on the tether, the rotation rate remains unchanged. This can also be examined in the framework of angular momentum.

$$H = I\omega$$

Although the moment of inertia of the tether station increases when the payload is caught on the end, the total angular momentum of the station increases commensurately, and the angular rate remains constant.

After catching a payload, the CM of the tether station will move to a location much closer to the payload. Recall that the CM location is the only point on the tether that follows a Keplerian orbit. This means that right before catching, the new CM location was in a *non-Keplerian* orbit, with a position and velocity (relative to the center of the Earth) that is quite different than the original CM.

To understand what the tether station's new orbit will be, the new CM's position and velocity must be used to propagate a new orbit. This new orbit will have less orbital energy than the previous orbit. The tether station has lifted the payload from a lower energy state to a higher one, at the expense of its own orbital energy. A similar process takes place when the payload is thrown, and the tether station drops to an even lower orbital energy. This is the heart of the concept of momentum exchange.

Advanced Orbital Analysis of Momentum Exchange: Establishing a Resonant, Repeating Orbit

As has been mentioned, the iterative nature of MXER tether design lends itself well to implementation in a spreadsheet. A spreadsheet was developed in MS Excel to enable the orbital parameters of the tether station and payload to be determined accurately and changed rapidly. The iterative capabilities of Excel enable equations that encompass circular references to be solved quickly and accurately. For instance, the mass of the tether is a function of the tether tip velocity, which is a function of the distance from the center of mass of the tether station, which is a function of the mass of the tether, and so forth.

An important consideration when considering a capture operation between a tether and its payload is the possibility it might fail. In such a case, the simplest way to recover from the failure is to configure the orbits so that they will meet again at a future time.

In a real-world mission scenario, many perturbative effects must be considered and modeled is order to accurately propagate an orbit:

- Non-ideal mass distribution of the Earth
- Lunar and solar perturbations
- Atmospheric drag effects
- Ionospheric and magnetic interactions with the spacecraft
- Solar radiation pressure

In this analysis, the only perturbative effect considered is the first gravitational harmonic of the Earth, often referred to as J2. J2 models the oblateness of the Earth, and it induces both secular (constant) and sinusoidal changes in three of the orbital elements of a spacecraft.

For a spacecraft in an equatorial elliptical orbit, the secular rate of change[4] in the longitude of the ascending node is given by the equation:

$$\dot{\Omega} = -\frac{3}{2} J_2 \frac{R_{\oplus}{}^2}{p^2} \bar{n}$$

Similarly, the secular rate of change of the argument of periapsis is given by:

$$\dot{\omega} = 3 J_2 \frac{R_{\oplus}{}^2}{p^2} \bar{n}$$

The rate of the change of the initial value for mean anomaly is given by:

$$\dot{M}_o = -\frac{3}{2} J_2 \frac{R_{\oplus}{}^2}{p^2} n \sqrt{1-e^2}$$

Since the initial value of mean anomaly is being shifted on each orbit, an accessory definition to orbital period and mean motion is necessary. For example, because of perturbations, one orbital period no longer defines the time that the spacecraft takes to go from periapsis to periapsis; instead it takes a little longer, as shown in Figure A-1.

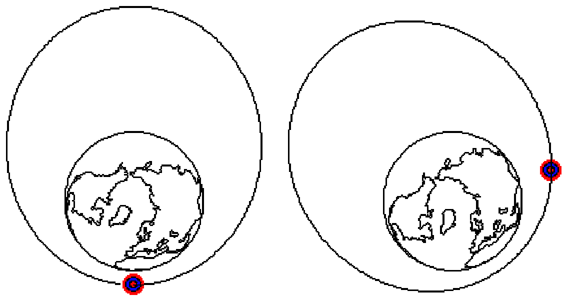

Figure A-1: The periapsis position is not reached after each orbital period due to perturbations. This shift must be accommodated for in calculations of the perturbed orbit.

The actual time between periapsis passages can be calculated from the following equation:

$$p' = \frac{2\pi}{\bar{n}}$$

While the difference between the orbital period of such a satellite and the time between periapsis passages may only amount to a few seconds, a few seconds for a tether travelling at 9 km/s will mean that the tether misses the payload by many tens of kilometers.

Mean motion is defined as the average angular distance (in radians) a spacecraft travels each second:

$$n = \sqrt{\frac{\mu}{a^3}}$$

The "mean" mean motion is defined as the mean motion minus the change is the initial value of mean anomaly:

$$\bar{n} = n - \dot{M}_0$$

This "mean" mean motion is used in calculating the rates of change in ascending node and apsidal location.

A resonant orbit can be defined between a tether in an elliptical equatorial orbit and a payload in a circular equatorial orbit by solving the following equality:

$$mp_2'\left(\dot{\Omega}_1 + \dot{\omega}_1 + \bar{n}_1\right) - \ell(2\pi) = mp_2'\left(\dot{\Omega}_2 + \dot{\omega}_2\right)$$

In this equality, m and □ represent the resonance ratio. For instance, if the payload completes 3 orbits for every 1 orbit of the tether station, m=3 and □=1.

Although this is a highly implicit equation, every variable term within it can be boiled down to four numbers, the perigee and apogee of the payload and tether. Further analysis shows that if the payload is in a circular orbit, its perigee and apogee will be the same, and if the tether is designed to catch the payload, the perigee of its CM will simply be the payload's perigee plus the tether arm length. This leaves only the tether's apogee to be solved, and the iteration features of Excel solve it quickly.

A spreadsheet that incorporates all these features in it has been developed and is available upon request to those who wish to conduct their own analyses. Please contact the author at kirk.sorensen@msfc.nasa.gov.

Appendix B

Techniques for Tether Mass Property Evaluation

Characteristic velocity is:

$$v_c = \sqrt{\frac{2T}{fd}}$$

Dividing tip velocity by characteristic velocity yields a velocity ratio:

$$VR = \frac{v_{tip}}{v_c}$$

In Moravec's derivation[7] of the mass of a tapering tether, he made several simplifying assumptions to enable the equation to be integrated.

- A uniform tether with uniform properties.
- A tether that connects two equal endmasses; hence, the center of mass of the system would be at the center of the tether.
- Centrifugal accelerations only (no gravitational interaction).

His equation for tether area as a function of radius (from the center of rotation of the system) is:

$$A(r) = \frac{m_{tip} {v_{tip}}^2 f}{\ell \tau} \exp\left(VR^2\left(1 - \frac{r^2}{\ell^2}\right)\right)$$

The area of the tether at the center of rotation (r = 0) is:

$$A(r) = \frac{m_{tip} {v_{tip}}^2 f}{\ell \tau} \exp\left(VR^2\right)$$

Similarly, the area of the tether at the tip (r = l) is:

$$A(r) = \frac{m_{tip} {v_{tip}}^2 f}{\ell \tau}$$

This can be integrated in the form:

$$m_{tether} = \rho \int_0^{\ell} A(r) dr$$

The mass ratio is:

$$MR = \frac{m_{tether}}{m_{tip}} = \sqrt{\pi}(VR)\exp\left(VR^2\right)erf(VR)$$

Since any terms containing reference to the length of the tether have dropped out of the expression, we can conclude that the mass of the tether is a function only of the mass at the tether tip, and the velocity ratio. Unfortunately, it is a function of the exponential of velocity ratio squared, which means that the tether mass will grow very quickly for increasing tip velocities.

In order to derive the center-of-mass of the tether segment, each differential mass element must be multiplied by its distance from the center of rotation. The result is then divided by the overall mass.

$$CM = \frac{\rho \int_0^{\ell} A(r) r dr}{\rho \int_0^{\ell} A(r) dr} = \frac{\rho \int_0^{\ell} A(r) r dr}{m_{tip}(MR)}$$

After integration and simplification, this yields:

$$CM = \frac{\ell}{MR}\left[\exp\left(VR^2\right) - 1\right]$$

Hence, it can be clearly seen that the location of the center-of-mass of the tether segment is essentially only a function of velocity ratio and segment length. This interesting result means that the properties of the tether are quite independent of tether geometry.